CIRCLES

Lines and Circles

Prepared by: Kenneth P. Montajes

Definition

A **circle** is a set of all points in a plane equidistant to a fixed point (center). The constant equal distance is the **radius** of the circle.

Definition

A **chord** is any line segment that connects two points of a circle.

Is the **diameter** of a circle a chord?

A **secant** is any line that intersects two points of a circle

Circle centered at the origin

The circle with center at the point $(0,0)$ and radius r has an equation

$$x^2 + y^2 = r^2$$

Theorem

The circle with center at the point (h, k) and radius r has an equation in standard form (center-radius)

$$(x - h)^2 + (y - k)^2 = r^2$$

Example

Find the equation of a circle having a diameter with endpoints at $A(-2,3)$ and $B(4,5)$. Plot the circle.

Example

Is the point $A(3,4)$ inside, outside, or on the circle with equation

$$(x + 2)^2 + (y - 3)^2 = 9$$

General Form

The general form of an equation of a circle is given by

$$x^2 + y^2 + Dx + Ey + F = 0$$

Example

Find the center and radius of a circle having the equation

$$x^2 + y^2 + 6x - 4y - 23 = 0$$

Example

Plot the circle by finding the x and y intercepts of the equation

$$x^2 + y^2 - 4x + 4y - 32 = 0$$

Symmetry of a Circle

A circle has all three types of symmetry.

The Tangent Line

In plane geometry, the tangent line at point P on the circle is the line intersecting the circle at only one point.

Example

Find an equation of the tangent line to the circle

$$x^2 + y^2 - 6x - 2y - 15 = 0$$

at the point $P(6,5)$.

Seatwork

1. Find an equation of the circle that has endpoints $P(1,8)$ and $Q(5,-6)$ as the endpoints of the diameter.

2. Find the area of the region that lies outside the circle $x^2 + y^2 = 4$ but inside the circle $x^2 + y^2 - 4y - 12 = 0$.

3. What is the equation of the tangent line to the circle $2x^2 + 2y^2 = 196$ at the point $(-7,-7)$?

References

- The Calculus 7 by Louis Leithold